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A probability constrained multi-objective optimization model for CCHP system operation decision support

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HIGHLIGHTS

- A multi-objective optimization model for combined heat and power is proposed.
- The model minimizes operational cost, primary energy consumption and CO_2 emissions.
- The proposed stochastic model guarantees the optimized system operation.
- An incentive model is presented to assist the multi-objective decision analysis.

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ABSTRACT

Due to its capability to reduce carbon dioxide emission and to increase energy efficiency, the combined cooling, heating, and power (CCHP) system has attracted great attention during the last decade. A large number of deterministic and stochastic optimization models have been proposed to study the CCHP operation strategy. However, fewer studies have been conducted to optimize CCHP operation simultaneously with multiple objectives such as minimizing operational cost, primary energy consumption (PEC) and carbon dioxide emissions (CDE) considering the reliability of the CCHP operation strategy. In this research, we propose a stochastic multi-objective optimization model to optimize the CCHP operation strategy for different climate conditions based on operational cost, PEC and CDE. The probability constraints are added into the stochastic model to guarantee the optimized CCHP operation strategy is reliable to satisfy the stochastic energy demand. The study shows that a higher reliability level of the probability constraint will increase the operational cost, PEC and CDE. To assist the multi-objective decision analysis, we developed an incentive model for PEC and CDE reduction. The analysis results demonstrate how the incentive values for PEC and CDE reduction can be effectively determined using the proposed model for different climate locations.

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1. Introduction

During the last decade, the combined cooling, heating, and power (CCHP) system has attracted great attention due to its potential to reduce carbon and air pollutant emissions and to increase resource energy efficiency dramatically [1–11]. CCHP could simultaneously satisfy the power demand by generating electrical power through reciprocating internal combustion engines, turbine engines, or fuel cells, and satisfy the heating and cooling load by recovering waste heat [1]. In order to efficiently dispatch the energy generated by CCHP, extensive research has been conducted to develop operation strategies for CCHP. The existing CCHP operation strategy development can be classified into two general categories: (1) simple rule based operation strategy, (2) optimal/near-optimal operation strategy. Other than efficiently operating the CCHP system, the performance of CCHP could also be improved by integrating with other renewable energy sources, such as biomass [10], and solar energy [12,13].

Two simple rule-based operation strategies have been developed to operate the CCHP according to either electrical or thermal demand. In the case of following the electric load (FEL) operation strategy, all the electrical demand must be supplied by the power generation unit (PGU) in CCHP. For the following the thermal load (FTL) strategy, the adequate recovered waste heat must be provided to satisfy the heating and cooling demand. The performance and effectiveness of these two operation strategies are studied in [14–16]. However, the FEL and FTL strategies underperform the operation strategies derived by optimization models in terms of cost savings and carbon emissions reductions. Therefore, several attempts are proposed to develop optimal/near-optimal operation strategies for CCHP by using optimization techniques. In general,

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Nomenclature

a	fuel-to-electric-energy conversion parameter
b	fuel-to-electric-energy conversion parameter
c_{el}	cost of electricity imported from the electric grid
c_{f_pgu}	cost of fuel used by the PGU
c_{f_boiler}	cost of fuel used by the boiler
CDE	carbon dioxide emission
CCHP	combined cooling, heating, and power
CHP	combined heating and power
ECF_{CDE}	emission conversion factor for electricity
ECF_{PEC}	site-to-energy conversion factor for electricity
FCF_{CDE}	emission conversion factor for fuel
FCF_{CDE}	emission conversion factor for fuel
FCF_{PEC}	site-to-energy conversion factor for fuel
FCF_{PEC}	site-to-energy conversion factor for fuel
f_{sto}	values of the utility function under the stochastic case
f_{det}	values of the utility function under the deterministic case
E_d	electric energy demand
E_{excess}	excess electric energy produced by the PGU
$E_{facility}$	electric energy provided to the building
E_{grid}	electric energy imported from the electric grid
E_{pgu}	PGU electric energy output
EG	electric grid
$Energy_{required}$	total energy required of the system
$Energy_{loss\_total}$	total energy loss of the system
$Energy_{loss\_pgu}$	energy loss of the PGU
$Energy_{loss\_boiler}$	energy loss of the boiler
$Energy_{loss\_c}$	energy loss of the CCHP cooling components
$Energy_{loss\_h}$	energy loss of the CCHP heating components
F_{boiler}	boiler fuel energy consumption
F_{pgu}	PGU fuel energy consumption
MOO	multi-objective optimization
PEC	primary energy consumption
PGU	power generation unit
Q_{boiler}	thermal energy produced by the boiler
Q_{cool_d}	cooling energy demand
Q_{cool}	thermal energy produced by the cooling components
Q_{heat_d}	heating energy demand
Q_{heat}	thermal energy produced by the heating components
Q_{rcv}	recovered waste heat from the PGU
r	reliability level
z	reliability index

Symbols

η_{pgu_th}	fuel-to-thermal-energy conversion efficiency of PGU
η_{boiler}	boiler thermal efficiency
η_{cool_comp}	cooling components thermal efficiency
η_{heat_comp}	heating components thermal efficiency
Δ	impact index
σ	standard deviation
μ	mean

two optimization models are studied in the existing research which are: (1) deterministic model [1–5,9,17–26], and (2) stochastic model [6–8,27,28].

In the deterministic model, all the information is assumed to be well known and randomness is not considered. Cho et al. [1] proposed a linear programming (LP) model to determine optimal operation strategy for the CCHP in terms of operational cost, primary energy consumption (PEC), and carbon dioxide emission (CDE) separately. The impacts of the different energy price policies are studied in [3] by using the particle swarm optimization (PSO) algorithm. An enumeration algorithm is employed to study the operation strategy of a new structure of the CCHP system with hybrid chillers [9]. A tri-commodity simplex algorithm is developed in [17] to study CCHP operation by minimizing the energy cost. Thorin et al. [18] developed a tool to study long-term optimization of CCHP system based on a mixed integer linear programming (MILP) model. Other than single objective model, multi-objective optimization models are explored to optimize the CCHP system in terms of energy and environmental benefits simultaneously. A multi-objective optimization (MOO) model was proposed in [4] to optimize the CCHP system simultaneously in terms of exergetic efficiency, total levelized cost rate of the system product, and the cost rate of environmental impact. The superior performance of CCHP system in terms of cheaper operational cost and smaller CO_2 emission is demonstrated in an urban area [5]. Two objectives, competing fuel cost and environmental impact, were studied using a multi-objective line-up competition algorithm to optimize the CCHP economic dispatch [19]. Through studying a multi-criteria model which considers the performance of CCHP in terms of primary energy saving, CO_2 emission reduction, and annual total cost saving, some insights are gained such as the FTL strategy is preferred in the cold area, and the FEL strategy is better for the building having stable thermal demand in mild climate zone [20]. The meta-heuristic algorithms such as PSO [21], and genetic algorithm (GA) [22,23] are also employed to study the multi-objective optimization model for CCHP operation. A matrix modeling for the CCHP system is proposed in [26] to optimize the operation strategy considering multiple evaluation criteria.

Another effort to study CCHP operation is the stochastic model which optimizes the performance of the CCHP system with uncertainties in energy demand and energy price. In the real world, the operation strategy derived in the deterministic condition may be infeasible or cost expensive, so the stochastic optimization model which could derive reliable operation strategy is needed. Li et al. [27] proposed a mix-integer nonlinear programming (MINLP) model to study the impacts of the average, uncertainty, and peaks of energy demands on economic performance of CCHP system. Smith et al. [6] studied uncertainties in the thermal load, natural gas prices, electricity prices, and engine performance, and investigated the performance of CCHP system in terms of operational cost, PEC, and CDE under these uncertainties. An uncertain programming model which integrates the Monte-Carlo method (MCM) and mixed-integer nonlinear programming was developed to derive optimized CCHP operation strategy under energy demand uncertainty [7]. Wang and Singh [8] developed a stochastic model for combined heating and power (CHP) system economic dispatch which could simultaneously optimize the performance of CCHP in terms of production cost, power generation deviation, and heat generation deviation, and propose an improved PSO algorithm to study the stochastic model.

The existing deterministic and stochastic models are demonstrated to be able to improve the performance of CCHP in terms of energy and environmental benefits. However, less study is conducted to optimize CCHP operation simultaneously with multiple objectives considering the reliability of the CCHP operation, such as how reliable the CCHP could satisfy the stochastic energy demand. In this research, we extend the deterministic model proposed in [1] as a multi-objective stochastic model with uncertain energy demand, and add probability constraints to guarantee the optimized operation strategy is reliable. A simplified equivalent stochastic model is further developed to conduct tradeoff analysis of the CCHP operation under different climate conditions. In order

to assist the multi-objective decision analysis, an incentive model for PEC and CDE reduction is proposed to evaluate the Pareto operation decisions derived from the stochastic model. The main contributions in this research are summarized as follows: (1) A stochastic model is formulated by introducing probability constraints, which could make the CCHP operation strategy more reliable. (2) The Pareto optimal operation strategies derived from the stochastic model allow us to conduct tradeoff analysis among multiple evaluation measures. (3) An incentive model is proposed to further investigate the benefits of CCHP in terms of primary energy savings and carbon dioxide emissions reductions.

This paper is organized as follows: Section 2 introduces the stochastic decision model and the solution strategy for the stochastic model; the case study for the CCHP operation decision using single objective stochastic model is presented in Section 3; followed by a study of multi-objective stochastic model and the incentive model in Section 4, and conclusions are drawn in Section 5.

2. Mathematical CCHP operation decision model

As described in [1], fuel is supplied to the PGU which could produce electric energy and reject heat as byproduct in the CCHP system. This electric energy is used to power building and operate auxiliary cooling and heating components. If the PGU does not produce enough electric energy to satisfy the electric demand, the difference can be imported from the electric grid (EG). If there is excess electricity, it can be exported or sold to the EG. The recovered waste heat from the PGU is used to produce cooling or heating to satisfy the building cooling and heating loads. If the heat recovered from the PGU is not enough to fulfill the thermal energy requirement for building space cooling and heating, a boiler is used to provide the remaining required heat. A network flow model (see Fig. 1) for the CCHP system is proposed in [1] based on the energy flow in CCHP system. A deterministic linear programming model is proposed to optimize the CCHP operation based on operational cost, PEC and CDE [1].

In this research, we consider the uncertainties in the energy demand (e.g., electric, cooling, and heating), and extend the deterministic linear programming as a stochastic model with some probability constraints to guarantee the derived operation strategy is reliable to satisfy the stochastic energy demand. The proposed stochastic model and its simplified equivalent formulation are described in the following sections.

2.1. Stochastic decision model I

In this model, we introduce three random parameters (E_d, Q_{cool_d}, Q_{heat_d}) to represent the uncertainties in the electric, cooling, and heating demand respectively, and add four probability constraints (see Eqs. (1)–(4)) to guarantee the CCHP operation will reliably satisfy the stochastic energy demand. The four probability constraints (Eqs. (1)–(4)) are introduced to guarantee the energy balance in nodes 1, 9, 10, and 11 in Fig. 1. The probability constraint indicates that the probability to satisfy the constraint should greater than a reliability level r.

$$P\{E_{grid}(t) + F_{pgu}(t) + F_{boiler}(t) - E_{excess}(t) - Energy_{loss\_total}(t) \geqslant E_d(t) + Q_{cool\_d}(t) + Q_{heat\_d}(t)\} \geqslant r_1 \quad (1)$$

$$P\{E_{facility}(t) \geqslant E_d(t)\} \geqslant r_2 \quad (2)$$

$$P\{Q_{cool}(t) \geqslant Q_{cool\_d}(t)\} \geqslant r_3 \quad (3)$$

$$P\{Q_{heat}(t) \geqslant Q_{heat\_d}(t)\} \geqslant r_4 \quad (4)$$

The decision variables used in this model are listed in Table 1. The lower bounds of all the decision variables are 0.

Three objective functions are studied in this model. The first objective (f_{cost}) is to minimize the total operational cost of running the CCHP system over T time periods, the second objective (f_{PEC}) is to minimize the total amount of primary energy consumption over

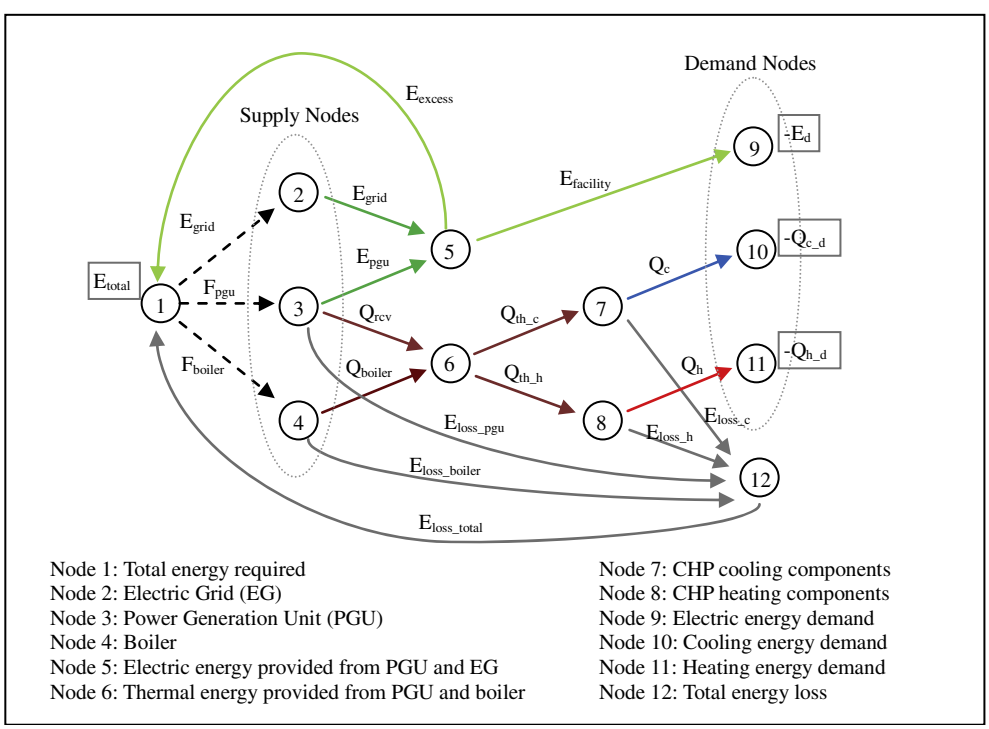

Fig. 1. Network flow model of a typical CCHP system.

Table 1
Decision variables (DVs) used in stochastic model I.

DVs	Descriptions
E_{grid}	Electric energy imported from the electric grid
F_{pgu}	PGU fuel energy consumption
F_{boiler}	Boiler fuel energy consumption
E_{pgu}	PGU electric energy output
Q_{rcv}	Recovered waste heat from the PGU
Q_{boiler}	Thermal energy produced by the boiler
$E_{facility}$	Electric energy consumption in the facility
Q_{th_cool}	Thermal energy required by the cooling components
Q_{th_heat}	Thermal energy required by the heating components
Q_{cool}	Thermal energy produced by the cooling components
Q_{heat}	Thermal energy produced by the heating components
E_{excess}	Excess electric energy produced by the PGU
$Energy_{loss\_pgu}$	Energy loss of the PGU
$Energy_{loss\_boiler}$	Energy loss of the boiler
$Energy_{loss\_c}$	Energy loss of the CCHP cooling components
$Energy_{loss\_h}$	Energy loss of the CCHP heating components
$Energy_{loss\_total}$	Total energy loss of the system

T time periods, and the last objective (f_{CDE}) is to minimize the total amount of carbon dioxide emissions over T time periods.

$$f_{cost} = \sum_{t=1}^{T}\{c_{el}E_{grid}(t) + c_{f\_pgu}F_{pgu}(t) + c_{f\_boiler}F_{boiler}(t)\} \quad (5)$$

$$f_{PEC} = \sum_{t=1}^{T}\{ECF_{PEC}E_{grid}(t) + FCF_{PEC\_pgu}F_{pgu}(t) + FCF_{PEC\_boiler}F_{boiler}(t)\} \quad (6)$$

$$f_{CDE} = \sum_{t=1}^{T}\{ECF_{CDE}E_{grid}(t) + FCF_{CDE\_pgu}F_{pgu}(t) + FCF_{CDE\_boiler}F_{boiler}(t)\} \quad (7)$$

where t is the time period index ($t = 1,\ldots,T$), c_{el}, c_{f_pgu}, and c_{f_boiler} represent the cost of purchasing 1 kW h of electricity, the cost of fuel that is used to produce 1 kW h of energy in the PGU, the cost of fuel that is used to produce 1 kW h of energy in the boiler respectively, ECF_{PEC}, and FCF_{PEC_pgu}, and FCF_{PEC_boiler} represent site-to-primary energy conversion factors for electricity, fuel used in PGU, and fuel used in boiler respectively, ECF_{CDE}, and FCF_{CDE_pgu}, and FCF_{CDE_boiler} represent emission conversion factors for electricity, fuel used in PGU, and fuel used in boiler respectively.

Constraints expressed in Eqs. (8)–(14) represent the energy balance in nodes 3, 4, 5, 6, 7, 8, and 12 in Fig. 1. The fuel-to-electric-energy conversion of a PGU is expressed in Eq. (15). Four energy efficiency constraints for PGU, boiler, cooling and heating component are presented in Eqs. (16)–(19).

$$E_{pgu}(t) + Q_{rcv}(t) + Energy_{loss\_pgu}(t) - F_{pgu}(t) = 0 \quad (8)$$

$$Q_{boiler}(t) + Energy_{loss\_boiler}(t) - F_{boiler}(t) = 0 \quad (9)$$

$$E_{excess}(t) + E_{facility}(t) - E_{grid}(t) - E_{pgu}(t) = 0 \quad (10)$$

$$Q_{th\_cool}(t) + Q_{th\_heat}(t) - Q_{rcv}(t) - Q_{boiler}(t) = 0 \quad (11)$$

$$Q_{cool}(t) + Energy_{loss\_c}(t) - Q_{th\_cool}(t) = 0 \quad (12)$$

$$Q_{heat}(t) + Energy_{loss\_h}(t) - Q_{th\_heat}(t) = 0 \quad (13)$$

$$Energy_{loss\_total}(t) - Energy_{loss\_pgu}(t) - Energy_{loss\_boiler}(t) - Energy_{loss\_c}(t) - Energy_{loss\_h}(t) = 0 \quad (14)$$

$$F_{pgu}(t) = \begin{cases} aE_{pgu}(t) + b & E_{pgu}(t) > 0 \\ 0 & E_{pgu}(t) = 0 \end{cases} \quad (15)$$

$$Q_{rcv}(t) - \eta_{pgu\_th}F_{pgu}(t) = 0 \quad (16)$$

$$Q_{boiler}(t) - \eta_{boiler}F_{boiler}(t) = 0 \quad (17)$$

$$Q_{cool}(t) - \eta_{cool\_comp}Q_{th\_cool}(t) = 0 \quad (18)$$

$$Q_{heat}(t) - \eta_{heat\_comp}Q_{th\_heat}(t) = 0 \quad (19)$$

where a and b are two energy fuel-to-electric-energy conversion parameters, η_{pgu_th}, η_{boiler}, η_{cool_comp}, and η_{heat_comp} are fuel-to-thermal-energy conversion efficiency of PGU, boiler thermal efficiency, cooling and heating components thermal efficiency.

2.2. Stochastic decision model II

It is observed that some decision variables and equality constraints in the stochastic decision model I could be removed by converting the equality constraints into inequality constraints. Therefore, we propose an equivalent stochastic decision model II to simplify the stochastic model I. In this model, we keep decision variables E_{grid}, F_{pgu}, F_{boiler}, $E_{facility}$, Q_{cool}, and Q_{heat}, and introduce a set of binary variables S_{pgu} to indicate the state of PGU during the T time periods where 1 denotes PGU is on and 0 denotes PGU is off. The objective functions are the same as the stochastic model I which are expressed in Eqs. (5)–(7). Constraints in Eqs. (1)–(4) and (8)–(19) are equivalent to the following constraints in Eqs. (20)–(27). The detail process to obtain these constraints is presented in Appendix A.

$$(1 + a\eta_{pgu\_th} - a)F_{pgu}(t) - bS_{pgu}(t) \leqslant 0 \quad (20)$$

$$aE_{facility}(t) - aE_{grid}(t) - F_{pgu}(t) + bS_{pgu}(t) \leqslant 0 \quad (21)$$

$$Q_{cool}(t)/\eta_{cool\_comp} + Q_{heat}(t)/\eta_{heat\_comp} - \eta_{pgu\_th}F_{pgu}(t) - \eta_{boiler}F_{boiler}(t) = 0 \quad (22)$$

$$F_{pgu}(t) \leqslant MS_{pgu}(t) \quad (23)$$

$$P\{E_{facility}(t) + Q_{cool}(t) + Q_{heat}(t) \geqslant E_d(t) + Q_{cool\_d}(t) + Q_{heat\_d}(t)\} \geqslant r_1 \quad (24)$$

$$P\{E_{facility}(t) \geqslant E_d(t)\} \geqslant r_2 \quad (25)$$

$$P\{Q_{cool}(t) \geqslant Q_{cool\_d}(t)\} \geqslant r_3 \quad (26)$$

$$P\{Q_{heat}(t) \geqslant Q_{heat\_d}(t)\} \geqslant r_4 \quad (27)$$

In this research, we assume the three stochastic energy demands at each time period t are independent and follow the normal distributions in which 95% of the whole area is within the range of ±20% of the average energy demands [7,27–29]. According to [30], the four probability constraints in Eqs. (24)–(27) are equivalent to four deterministic constraints in Eqs. (28)–(31) when the random parameters in the probability constraints are normal distributions.

$$E_{facility}(t) + Q_{cool}(t) + Q_{heat}(t) \geqslant Z_1(t) \quad (28)$$

$$E_{facility}(t) \geqslant Z_2(t) \quad (29)$$

$$Q_{cool}(t) \geqslant Z_3(t) \quad (30)$$

$$Q_{heat}(t) \geqslant Z_4(t) \quad (31)$$

$$Z_1(t) = \mu_{E_d}(t) + \mu_{Q_{cool\_d}}(t) + \mu_{Q_{heat\_d}}(t) + z_{\alpha_1}\sqrt{\sigma^2_{E_d}(t) + \sigma^2_{Q_{cool\_d}}(t) + \sigma^2_{Q_{heat\_d}}(t)} \quad (32)$$

$$Z_2(t) = \mu_{E_d}(t) + z_{\alpha_2}\sigma_{E_d}(t) \quad (33)$$

$$Z_3(t) = \mu_{Q_{cool\_d}}(t) + z_{\alpha_3}\sigma_{Q_{cool\_d}}(t) \quad (34)$$

$$Z_4(t) = \mu_{Q_{heat\_d}}(t) + z_{\alpha_4}\sigma_{Q_{heat\_d}}(t) \quad (35)$$

where $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$, and Z_{α_4} are reliability index corresponding to the reliability level r_1, r_2, r_3, and r_4, $Z_{\alpha_1} = \Phi^{-1}(r_i), i = 1,\ldots,4$, and Φ^{-1} is the inverse transformation of the standard normal distribution function.

2.3. Solution strategy of the CCHP operation decision model

It is assumed that there is no interaction between variables at different time periods in [1], therefore the stochastic model presented in Section 2.2 could be decomposed as T independent sub-problems which are mixed binary linear programming problems. In this research, we propose a utility function to combine the three objective functions which is expressed as

$$\min f(t) = c_{grid}(t)E_{grid}(t) + c_{pgu}(t)F_{pgu}(t) + c_{boiler}(t)F_{boiler}(t) \tag{36}$$

where c_{grid}, c_{pgu}, and c_{boiler} are the coefficient corresponding to the electricity, fuel used in PGU and fuel used in boiler. For example, $c_{grid} = c_{el}$, $c_{pgu} = c_{f\_pgu}$, and $c_{boiler} = c_{f\_boiler}$ if the utility function is defined to minimize the operational cost. In order to solve the sub-problem at each time period t, we decomposed the problem into two separate linear programming problems: one problem is in the case that PGU is off ($S_{pgu}(t) = 0$), and the other problem is in the case that PGU is on ($S_{pgu}(t) = 1$). The optimal solution could be determined as the case which minimizes the utility function expressed in Eq. (36).

2.3.1. Case I: PGU is off

In this case, the PGU is off which means $S_{pgu}(t) = 0$, and $F_{pgu}(t) = 0$. Therefore the linear programming model at time period t could be simplified as,

$$\min f(t) = c_{grid}(t)E_{grid}(t) + c_{boiler}(t)F_{boiler}(t) \tag{37}$$
$$E_{facility}(t) - E_{grid}(t) \leqslant 0 \tag{38}$$
$$Q_{cool}(t)/\eta_{cool\_comp} + Q_{heat}(t)/\eta_{heat\_comp} - \eta_{boiler}F_{boiler}(t) = 0 \tag{39}$$
$$E_{facility}(t) + Q_{cool}(t) + Q_{heat}(t) \geqslant Z_1(t) \tag{40}$$
$$E_{facility}(t) \geqslant Z_2(t) \tag{41}$$
$$Q_{cool}(t) \geqslant Z_3(t) \tag{42}$$
$$Q_{heat}(t) \geqslant Z_4(t) \tag{43}$$

It is observed that the constraint in Eq. (38) must be satisfied in the optimal solution. The optimal solution for this linear programming model is

$$E_{facility}(t) = \begin{cases} \max(Z_1(t) - Z_3(t) - Z_4(t), Z_2(t)) & \text{if } c_{grid}(t)\eta_{boiler}/c_{boiler}(t) \leqslant \min(1/\eta_{cool\_comp}, 1/\eta_{heat\_comp}) \\ Z_2(t) & \text{otherwise} \end{cases}$$
$$Q_{cool}(t) = \begin{cases} \max(Z_1(t) - Z_2(t) - Z_4(t), Z_3(t)) & \text{if } 1/\eta_{cool\_comp} \leqslant \min(c_{grid}(t)\eta_{boiler}/c_{boiler}(t), 1/\eta_{heat\_comp}) \\ Z_3(t) & \text{otherwise} \end{cases} \tag{44}$$
$$Q_{heat}(t) = \begin{cases} \max(Z_1(t) - Z_2(t) - Z_3(t), Z_4(t)) & \text{if } 1/\eta_{heat\_comp} \leqslant \min(1/\eta_{cool\_comp}, c_{grid}(t)\eta_{boiler}/c_{boiler}(t)) \\ Z_4(t) & \text{otherwise} \end{cases}$$

2.3.2. Case II: PGU is on

In this case, the PGU is on which means $S_{pgu}(t) = 1$. Therefore the linear programming model could be simplified as

$$\min f(t) = c_{grid}(t)E_{grid}(t) + c_{pgu}(t)F_{pgu}(t) + c_{boiler}(t)F_{boiler}(t) \tag{45}$$
$$(1 + a\eta_{pgu\_th} - a)F_{pgu}(t) - b \leqslant 0 \tag{46}$$
$$aE_{facility}(t) - aE_{grid}(t) - F_{pgu}(t) + b \leqslant 0 \tag{47}$$
$$Q_{cool}(t)/\eta_{cool\_comp} + Q_{heat}(t)/\eta_{heat\_comp} - \eta_{pgu\_th}F_{pgu}(t) - \eta_{boiler}F_{boiler}(t) = 0 \tag{48}$$
$$E_{facility}(t) + Q_{cool}(t) + Q_{heat}(t) \geqslant Z_1(t) \tag{49}$$
$$E_{facility}(t) \geqslant Z_2(t) \tag{50}$$
$$Q_{cool}(t) \geqslant Z_3(t) \tag{51}$$
$$Q_{heat}(t) \geqslant Z_4(t) \tag{52}$$

A linear programming problem is solved to obtain an optimal solution in the case that PGU is on at time period t.

3. Single objective CCHP operation decision analysis

In this section, we will study the CCHP operation decision under energy demand uncertainty by using the stochastic model proposed in Section 2.2. In this research, we studied five cities (e.g., Columbus, MS; Minneapolis, MN; San Francisco, CA; Boston, MA; and Miami, FL) with different building electric, cooling, and heating load profiles. The hourly electric, cooling, and heating loads are obtained from a reference office building in building energy simulation software, EnergyPlus.[1] The detail information for the building settings are provided in [1]. The prices for electricity [31] and natural gas [32] (see Eq. (5)) for the studied cities, the site-to-primary energy conversion factors (see Eq. (6)) for purchased electricity and natural gas used to evaluate the PEC [33], and the CDE conversion factors (see Eq. (7)) for electricity [34] and natural gas [35] for the studied cities are listed in Table 2. The values of the coefficient used in Eqs. (15)–(19) are shown in [1]. According to [7,27–29], the electric, cooling, heating demand at each time period t are independent normal distributions. The mean value μ of the normal distribution is the value of each time period in the load profile, and 1.96σ (σ is the standard deviation of the normal distribution) equals 20% of μ [7,27–29]. The time period T in this study is 8760 h (1 year).

The selections of the reliability levels depend on the importance of the probability constraints. In this research, we assume all the four probability constraints are equal important and set equal reliability level for the four probability constraints expressed in Eqs. (24)–(27) which means that $r_1 = r_2 = r_3 = r_4 = r$, and optimize the three objective functions in Eqs. (5)–(7) separately with different reliability level r which chooses from the set [0.2, 0.5, 0.8, 0.95, 0.99]. In the future study, we will investigate the impacts of the unequal reliability level on the operational strategy. In practice, the reliability level is usually set to be greater than 0.8. In this study, the values which are less than 0.8 are chosen for a complete study of the influences of reliability levels on the performance of operational strategy. The optimal operational cost (f_{cost}), PEC (f_{PEC}), and CDE (f_{CDE}) for the five cities under different reliability levels are presented in Fig. 2.

For all the five cities, it is observed that the operational cost, PEC and CDE will increase as the reliability level is increased since increasing the reliability level is equivalent to increasing the energy demand (see Eqs. (28)–(35)). The condition $r = 0.5$ is equal to the deterministic case that $\sigma_{E_d}(t) = 0, \sigma_{Q_{cool\_d}}(t) = 0, \sigma_{Q_{heat\_d}}(t) = 0$ in Eqs. (32)–(35). The operational cost, PEC and CDE are less than the deterministic case when $r < 0.5$, and they are greater than the deterministic case when $r > 0.5$.

To further investigate the impact of reliability level to the oper-

[1] Detailed information of EnergyPlus is available at http://www.eere.energy.gov/buildings/energyplus/.

Table 2
Price, site-to-primary energy conversion factor, and CDE conversion factor used in the simulation.

City	Price ($/kW h)		Site-to-primary energy conversion factor		CDE conversion factor (tons/year-kW h)	
	Electricity	Natural gas	Electricity	Natural gas	Electricity	Natural gas
Columbus, MS	0.0948	0.027	3.340	1.047	0.000554	0.000184
Minneapolis, MN	0.0863	0.025			0.000703	0.000184
San Francisco, CA	0.1305	0.028			0.000279	0.000184
Boston, MA	0.1433	0.039			0.000560	0.000184
Miami, FL	0.0985	0.037			0.000599	0.000184

ational cost, PEC and CDE, we propose an impact index (Δ):

$$\Delta = |f_{sto} - f_{det}|/f_{det} \tag{53}$$

where f_{sto} and f_{det} are the values of the utility function in Eq. (36) under the stochastic and deterministic cases, respectively. The percentage values of the impact index of operational cost, PEC, and CDE for the five cities under different reliability levels are summarized in Table 3. The column "NG/E" is calculated as: (1) price of natural gas divided by the price of electricity for row "Cost", (2) site-to-primary energy conversion factor of natural gas divided by site-to-primary energy conversion factor of electricity for row "PEC", (3) CDE conversion factor of natural gas divided by CDE conversion factor of electricity for row "CDE".

For the cities of Columbus, San Francisco, and Boston, the reliability level r has the largest impact on CDE, and smallest impact on operational cost. The reliability level r has the largest impact on PEC, and smallest impact on CDE for the city of Minneapolis. Regarding the city of Miami, the reliability level r has the largest impact on operational cost, and the smallest impact on CDE. Increasing the reliability level r will increase the energy demand and thus increase the utilization of natural gas to satisfy heating and cooling demand. It indicates that changing electricity to natural gas will have the largest impact to increase CDE for the cities of Columbus, San Francisco, and Boston, to increase PEC for the city of Minneapolis, and to increase operational cost for the city of Boston. It follows the same pattern as the "NG/E" factor demonstrated in Table 3.

In summary, increasing the reliability level r for the probability constraint is equivalent to increasing the energy demand which will increase operational cost, PEC and CDE for all the five cities. The cities of Columbus, San Francisco, and Boston are the most sensitive in terms of CDE, least sensitive in terms of operational cost to the changing of reliability level (energy demand), while Miami is impacted in an opposite way. Minneapolis is more sensitive in terms of PEC, and less sensitive in terms of CDE.

4. Multi-objective CCHP operation decision analysis

In this section, we study the performance of CCHP operation decision by using a multi-objective model. The analysis results for the multi-objective stochastic decision model are presented in Section 4.1, followed by an incentive model based decision analysis for PEC and CDE reduction in Section 4.2

4.1. Multi-objective decision model

A weighted sum utility function is employed to consider the three objective functions together which is defined as

$$\min f_{ws} = w_1 f_{cost}/1000 + w_2 f_{PEC}/100000 + w_3 f_{CDE}/10 \tag{54}$$

where w_1, w_2, w_3 are the weights for operational cost, PEC and CDE, and $w_1 + w_2 + w_3 = 1$. We scale down the three objective functions to guarantee they are at the same magnitudes. To obtain the full range of Pareto solutions, the weights (w_1, w_2, w_3) need to be exhaustively enumerated. In this research, we change the weights in a 0.05 step size to enumerate the weights combinations, and the dominated solutions obtained from the weighted sum problem are removed. The Pareto frontier under five different reliability levels (r = [0.2, 0.5, 0.8, 0.95, 0.99]) for the five cities are obtained in this research. For demonstration, we show the Pareto frontier at the five reliability levels for San Francisco in Fig. 3. It is shown that the Pareto frontier at large reliability level is dominated by the Pareto frontier at small reliability level. The other four cities follow the same pattern as San Francisco. Due to the fact that increasing the reliability level is equivalent to increasing the energy demand, thus increasing the reliability level will increase the operational cost, PEC and CDE.

4.2. Incentive model based CCHP operation decision analysis

Multi-objective optimization can be effectively used in energy and environmental policy making, energy planning and resource allocation [36,37]. In this section, the proposed multi-objective model is used to assist the energy and environmental policy makers to promote the CCHP systems by creating incentives for the PEC and CDE reduction during its operations. To determine optimal incentives, we propose an incentive model for the PEC and CDE reduction to minimize the total cost f_{total} which is computed as

$$f_{total} = f_{cost} + p_{PEC}(f_{PEC} - f_{PEC\_ref}) + p_{CDE}(f_{CDE} - f_{CDE\_ref}) \tag{55}$$

where f_{PEC_ref} and f_{CDE_ref} are the PEC and CDE for the reference case, p_{PEC} ($/kW h) and p_{CDE} ($/ton) are the incentive for the PEC and CDE reduction. The Pareto frontiers for the five cities at 95% reliability level are chosen for decision analysis in the following sections.

4.2.1. Case I: incentive for primary energy saving

In this section, we only consider the incentive for primary energy saving which means $p_{PEC} \geqslant 0$ and $p_{CDE} = 0$. According to [38], the average retail price of electricity in the United States is approximately $0.10 per kW h, thus we run the incentive model by choosing p_{PEC} from the set [0, 0.02, 0.04, 0.06, 0.08, 0.1]. Thus the maximum incentive for PEC does not exceed the average retail price of electricity. The total cost, operational cost, PEC and CDE for reference case and different p_{PEC} values are presented in Table 4.

For the cities of Columbus, Boston, and Miami, increasing the incentive p_{PEC} for primary energy saving will decrease the total cost expressed in Eq. (55), PEC, and CDE, and increase the operational cost. For the city of San Francisco, PEC and CDE will be decreased and operational cost will be increased when the incentive p_{PEC} is increased. Increasing incentive p_{PEC} will decrease total cost and PEC, and increase operational cost and CDE for the city of Minneapolis.

4.2.2. Case II: incentive for carbon dioxide emission reduction

In this section, we only consider the incentive for carbon dioxide emission reduction which means $p_{PEC} = 0$ and $p_{CDE} \geqslant 0$. According to [39], the carbon emission price is approximated as

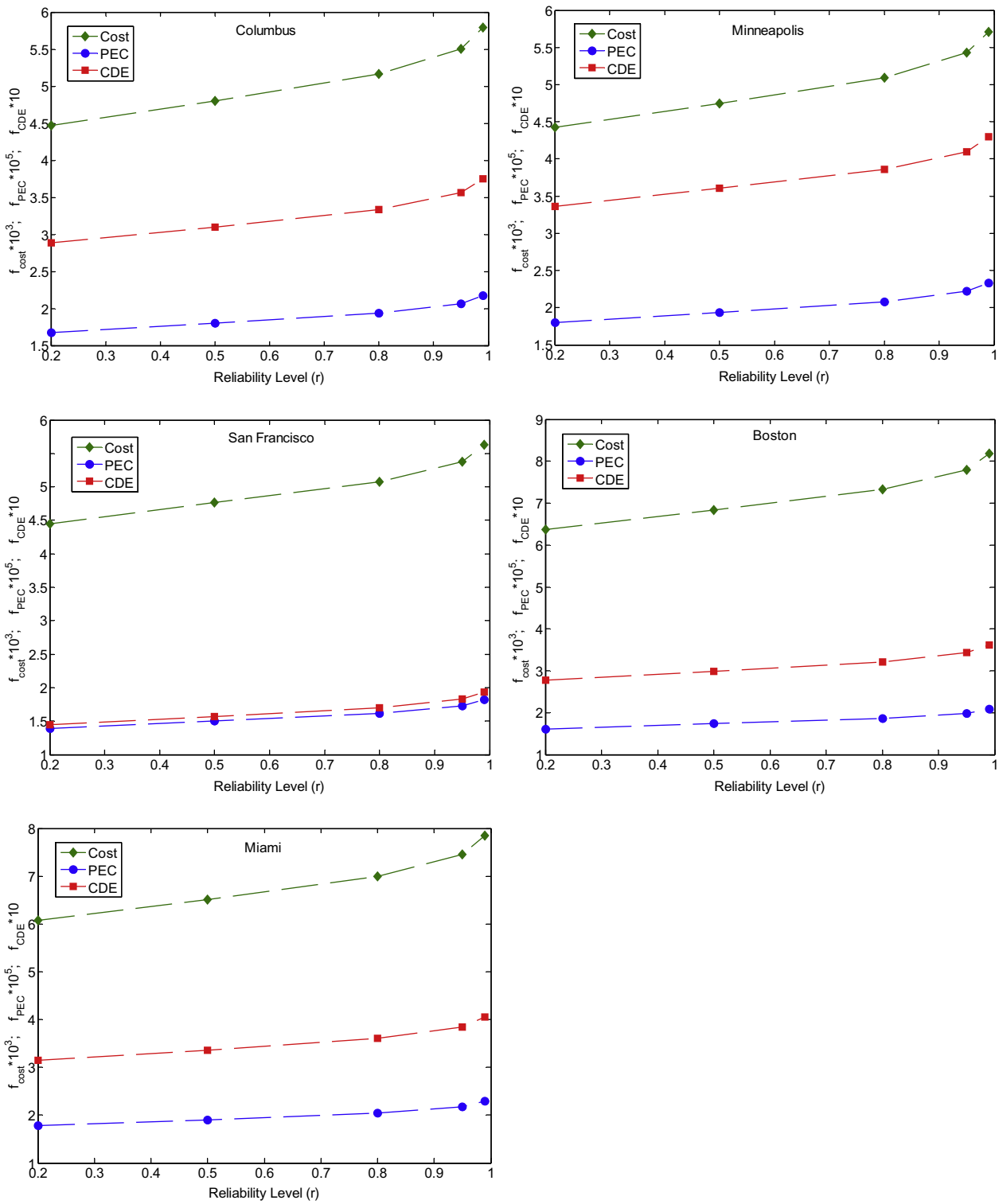

Fig. 2. Operational cost, PEC, and CDE under different reliability levels for the five cities.

$30 per ton, thus we run the incentive model by choosing p_{CDE} from the set [0, 10, 20, 30, 40, 50]. The total cost, operational cost, PEC and CDE for reference case and different p_{CDE} values are presented in Table 5.

For the city of Columbus, increasing the incentive p_{CDE} for carbon emission reduction will decrease the PEC and CDE, and increase the operational cost. For the city of Minneapolis, increasing the incentive p_{CDE} for carbon emission reduction will decrease the total cost and CDE, and increase the operational cost and PEC. For the city of San Francisco, PEC and CDE will be decreased, total cost and operational cost will be increased when the incentive p_{CDE} is increased. For the cities of Boston and Miami, increasing the incentive p_{CDE} for carbon emission reduction will decrease the total cost, PEC, and CDE, and increase the operational cost. It is always worse to use CCHP in economic aspect for the cities of Miami even if $50 per ton of CDE reduction incentive is considered. It is always better

Table 3
Impact index (%) for different reliability levels for the five cities.

City	r	0.2	0.5	0.8	0.95	0.99	Average	NG/E
Columbus, MS	Cost	6.867	0.000	7.467	14.551	20.533	9.884	0.2848
	PEC	6.904	0.000	7.580	14.778	20.860	10.024	0.3135
	CDE	6.925	0.000	7.643	14.898	21.028	10.099	0.3321
Minneapolis, MN	Cost	6.835	0.000	7.359	14.341	20.236	9.754	0.2897
	PEC	6.889	0.000	7.464	14.527	20.496	9.875	0.3135
	CDE	6.714	0.000	7.046	13.716	19.370	9.369	0.2617
San Francisco, CA	Cost	6.640	0.000	6.657	12.899	18.102	8.860	0.2146
	PEC	7.422	0.000	7.666	14.923	21.045	10.211	0.3135
	CDE	7.351	0.000	8.588	16.784	23.738	11.292	0.6595
Boston, MA	Cost	6.884	0.000	7.211	13.955	19.560	9.522	0.2722
	PEC	7.013	0.000	7.464	14.520	20.475	9.895	0.3135
	CDE	7.056	0.000	7.550	14.696	20.720	10.004	0.3286
Miami, FL	Cost	6.492	0.000	7.546	14.717	20.785	9.908	0.3756
	PEC	6.455	0.000	7.416	14.483	20.477	9.766	0.3135
	CDE	6.452	0.000	7.405	14.466	20.453	9.755	0.3072

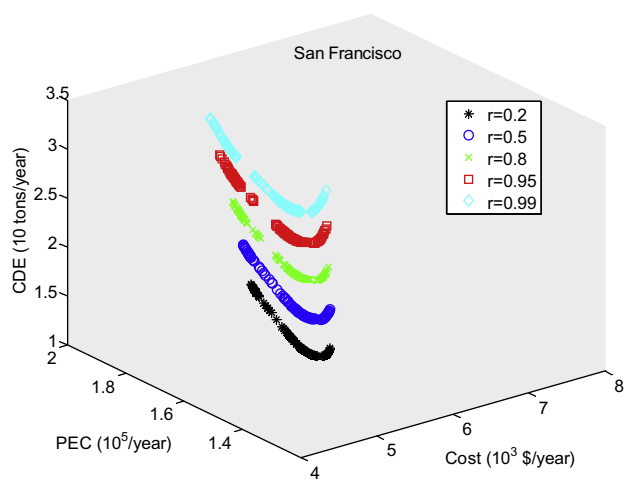

Fig. 3. Pareto frontiers under five reliability levels for San Francisco.

to use CCHP for the cities of Columbus, Minneapolis, San Francisco and Boston although no incentive for CDE is introduced.

4.2.3. Case III: incentive for PEC and CDE reduction

It is demonstrated that increasing the incentive p_{PEC} will decrease PEC, and increase operational cost, increasing the incentive p_{CDE} will decrease CDE and increase operational cost. In this section, we will change both the incentive p_{PEC} and p_{CDE}, and analyze the impacts of these incentives on total cost, operational cost, PEC and CDE. We run the incentive model by choosing p_{PEC} from the set [0.02, 0.04, 0.06, 0.08, 0.1], and p_{CDE} from the set [10, 20, 30, 40, 50]. The total cost, operational cost, PEC, and CDE for the five cities at reliability level 95% under different incentive combinations are recorded in Figs. 4–8 for the five cities.

For the city of Columbus, it is observed that increasing PEC incentive p_{PEC} and/or increasing CDE incentive p_{CDE} will decrease total cost, PEC, and CDE, and increase operational cost. One can always make CCHP system economically beneficial compared to reference case (i.e., conventional system) even without incentive for PEC saving and CDE reduction.

Table 4
Total cost, operational cost, PEC and CDE for different p_{PEC} values.

City		Reference	p_{PEC} = 0	p_{PEC} = 0.02	p_{PEC} = 0.04	p_{PEC} = 0.06	p_{PEC} = 0.08	p_{PEC} = 0.1
Columbus, MS	f_{total}	6047.9	5502.3	5351.3	5197.2	5042.4	4887.2	4731.8
	f_{cost}	6047.9	5502.3	5504.5	5506.4	5507.6	5508.2	5508.8
	f_{PEC}	214,528	207,192	206,864	206,798	206,775	206,764	206,758
	f_{CDE}	35.740	35.806	35.708	35.683	35.672	35.667	35.663
Minneapolis, MN	f_{total}	6064.2	5427.1	5082.9	4736.7	4390.0	4043.2	3696.3
	f_{cost}	6064.2	5427.1	5428.8	5429.7	5430.3	5430.7	5430.7
	f_{PEC}	239,181	222,132	221,886	221,855	221,842	221,836	221,836
	f_{CDE}	48.291	41.219	41.309	41.333	41.347	41.356	41.356
San Francisco, CA	f_{total}	6804.3	5377.7	5444.2	5477.5	5493.9	5451.1	5379.9
	f_{cost}	6804.3	5377.7	5405.1	5418.4	5524.3	5720.4	5757.5
	f_{PEC}	176,699	183,599	178,654	178,176	176,193	173,333	172,923
	f_{CDE}	15.505	28.768	26.704	26.426	24.888	22.347	21.916
Boston, MA	f_{total}	8851.3	7795.0	7570.5	7330.6	7087.4	6842.9	6597.7
	f_{cost}	8851.3	7795.0	7807.7	7814.8	7818.7	7823.0	7825.3
	f_{PEC}	211,103	201,449	199,244	198,999	198,914	198,852	198,827
	f_{CDE}	35.688	34.987	34.464	34.398	34.373	34.353	34.343
Miami, FL	f_{total}	6731.5	7460.2	7265.1	7066.0	6865.7	6664.7	6463.4
	f_{cost}	6731.5	7460.2	7463.3	7465.6	7467.9	7469.2	7470.0
	f_{PEC}	228,258	219,618	218,345	218,267	218,221	218,200	218,192
	f_{CDE}	40.936	38.781	38.528	38.511	38.500	38.495	38.493

Table 5
Total cost, operational cost, PEC and CDE for different p_{CDE} values.

City		Reference	p_{CDE} = 0	p_{CDE} = 10	p_{CDE} = 20	p_{CDE} = 30	p_{CDE} = 40	p_{CDE} = 50
Columbus, MS	f_{total}	6047.9	5502.3	5502.8	5503.1	5503.1	5503.1	5502.9
	f_{cost}	6047.9	5502.3	5502.5	5502.7	5503.1	5503.4	5503.9
	f_{PEC}	214,528	207,192	207,071	207,012	206,964	206,935	206,898
	f_{CDE}	35.740	35.806	35.773	35.755	35.741	35.732	35.719
Minneapolis, MN	f_{total}	6064.2	5427.1	5356.3	5285.4	5214.4	5143.2	5072.0
	f_{cost}	6064.2	5427.1	5427.2	5427.3	5427.5	5427.9	5428.3
	f_{PEC}	239,181	222,132	222,213	222,248	222,295	222,371	222,431
	f_{CDE}	48.291	41.219	41.201	41.194	41.186	41.174	41.166
San Francisco, CA	f_{total}	6804.3	5377.7	5504.4	5624.3	5740.6	5853.0	5963.8
	f_{cost}	6804.3	5377.7	5382.2	5388.4	5398.0	5406.5	5412.7
	f_{PEC}	176,699	183,599	180,920	179,902	179,100	178,589	178,337
	f_{CDE}	15.505	28.768	27.727	27.298	26.926	26.670	26.529
Boston, MA	f_{total}	8851.3	7795.0	7787.3	7778.5	7768.7	7757.6	7746.0
	f_{cost}	8851.3	7795.0	7795.6	7797.3	7800.3	7803.4	7805.1
	f_{PEC}	211,103	201,449	200,888	200,435	199,940	199,569	199,415
	f_{CDE}	35.688	34.987	34.858	34.751	34.634	34.544	34.507
Miami, FL	f_{total}	6731.5	7460.2	7437.3	7413.7	7390.0	7366.3	7342.4
	f_{cost}	6731.5	7460.2	7460.8	7460.9	7461.2	7461.4	7461.8
	f_{PEC}	228,258	219,618	218,630	218,587	218,519	218,486	218,443
	f_{CDE}	40.936	38.781	38.585	38.577	38.563	38.557	38.548

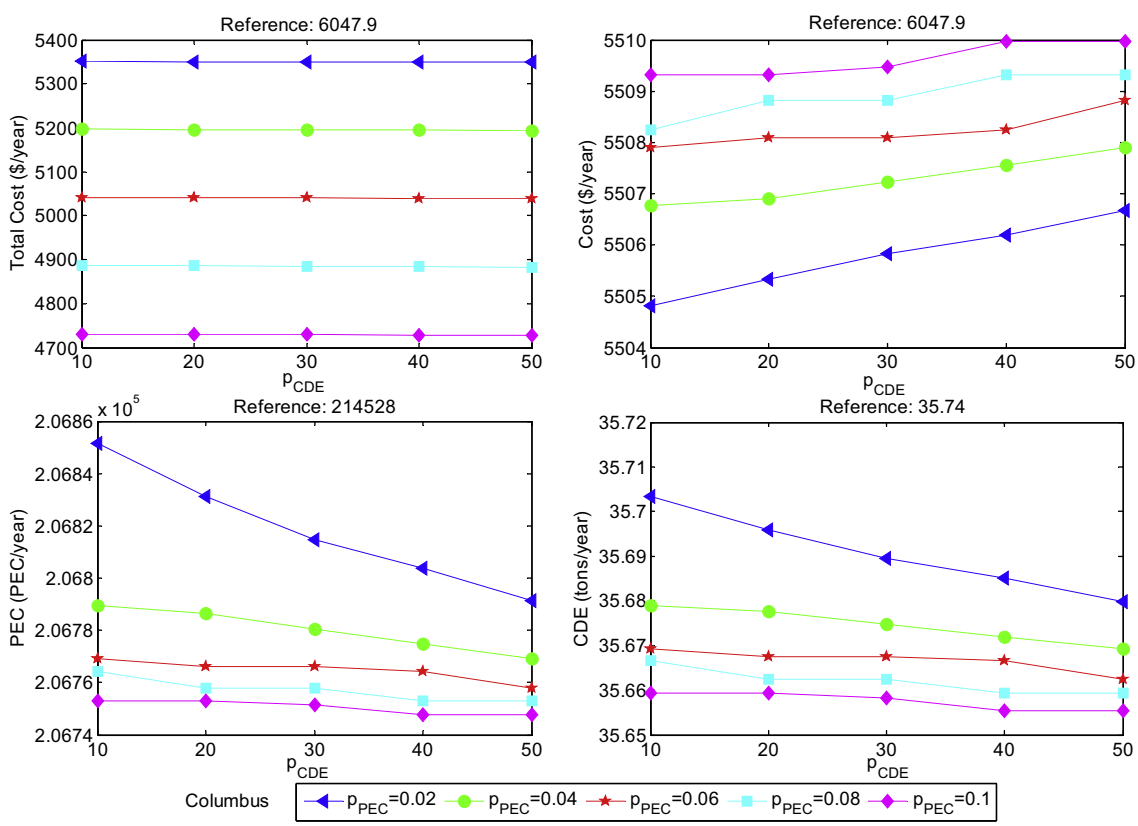

Fig. 4. Total cost, operational cost, PEC and CDE under different p_{PEC} and p_{CDE} values for Columbus.

For the city of Minneapolis, it is observed that increasing PEC incentive p_{PEC} will decrease total cost and PEC, and increase operational cost and CDE; increasing CDE incentive p_{CDE} will decrease total cost, operational cost and CDE, and increase PEC. Same as the Columbus case, CCHP system can be always cost effective even without incentive for PEC saving and CDE reduction.

For the city of San Francisco, it is observed that increasing PEC incentive p_{PEC} and/or increasing CDE incentive p_{CDE} will decrease PEC and CDE, and increase operational cost. Increasing incentive p_{CDE} will increase total cost. It is always better to use CCHP in economic aspect for the city of San Francisco and no incentive is necessarily required to promote CCHP in this city.

For the city of Boston, it is observed that increasing PEC incentive p_{PEC} and/or increasing CDE incentive p_{CDE} will decrease total cost, PEC and CDE, and increase operational cost. Same as the San Francisco case, it is always better to use CCHP economically for the city of Boston. CCHP is still economically feasible without any incentives for PEC and CDE.

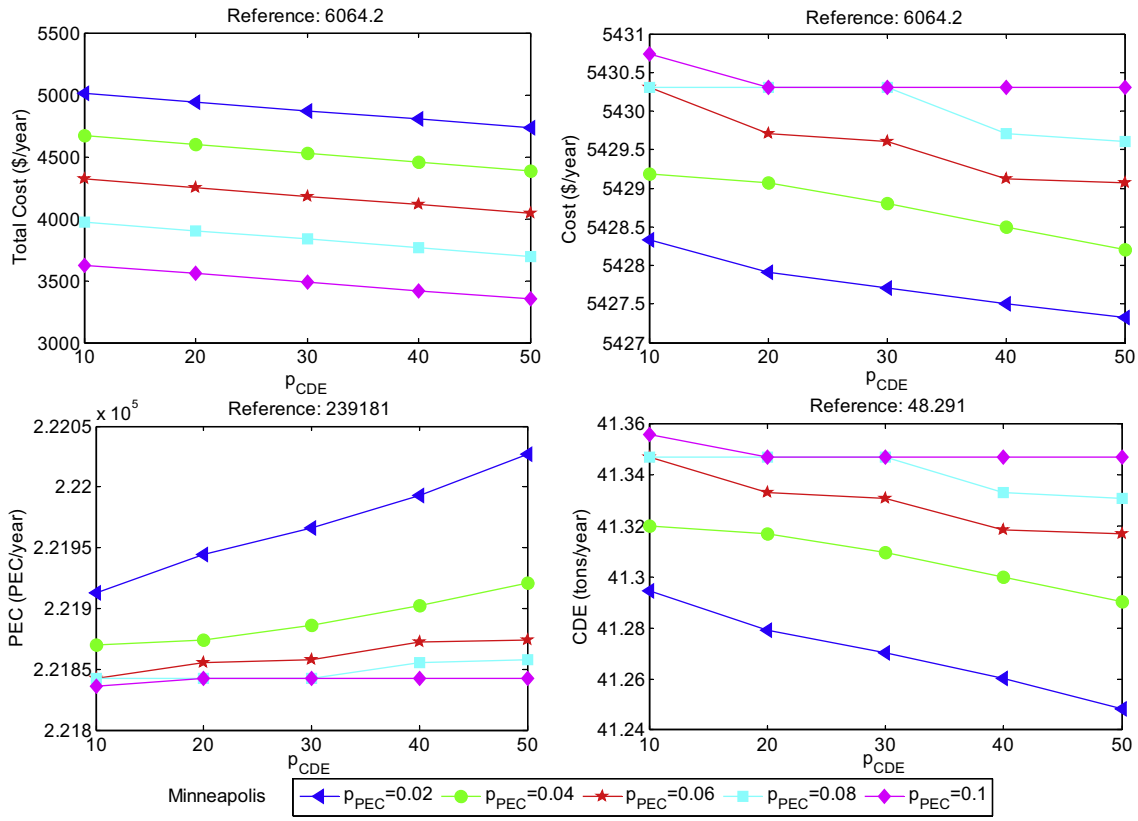

Fig. 5. Total cost, operational cost, PEC and CDE under different p_{PEC} and p_{CDE} values for Minneapolis.

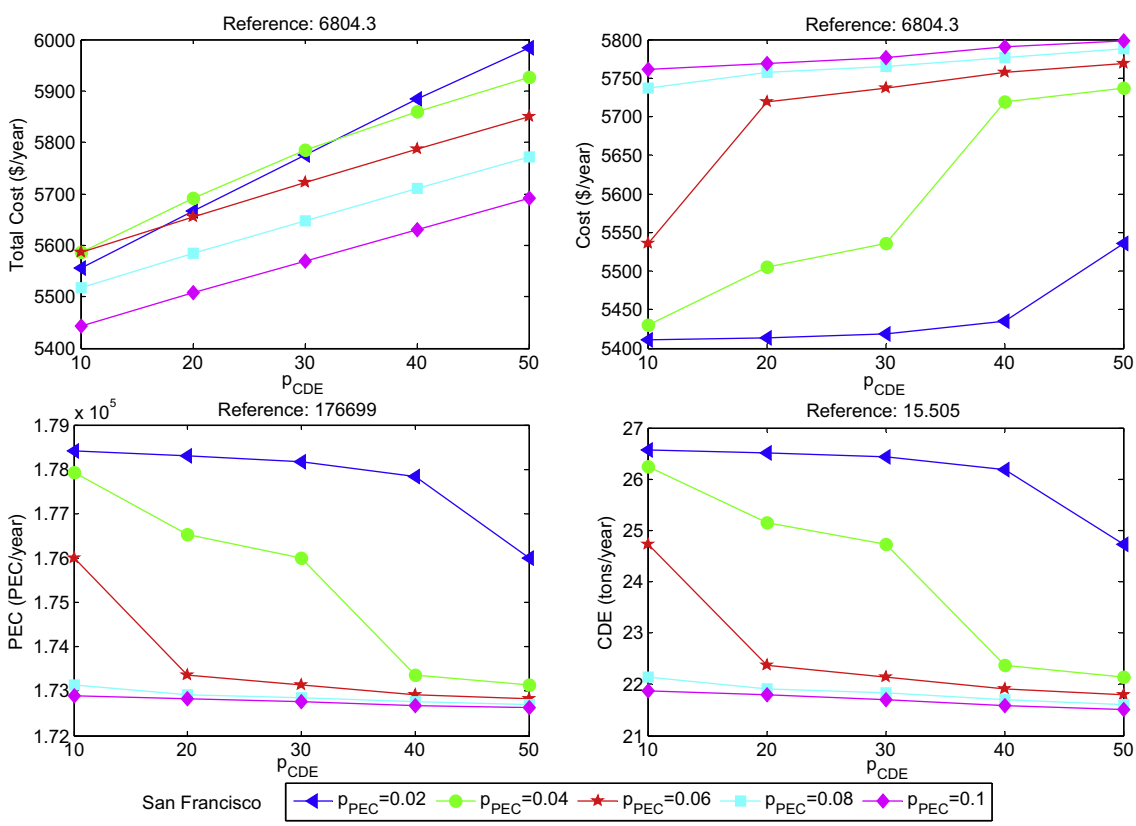

Fig. 6. Total cost, operational cost, PEC and CDE under different p_{PEC} and p_{CDE} values for San Francisco.

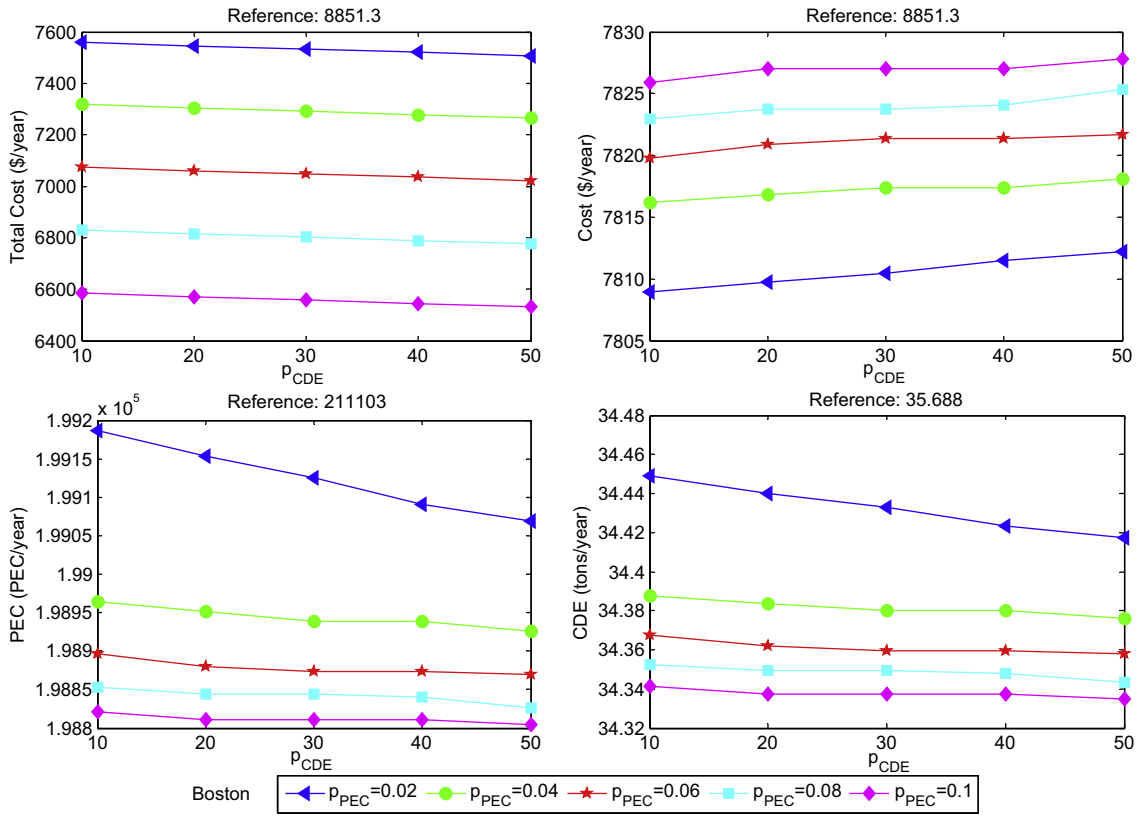

Fig. 7. Total cost, operational cost, PEC and CDE under different p_{PEC} and p_{CDE} values for Boston.

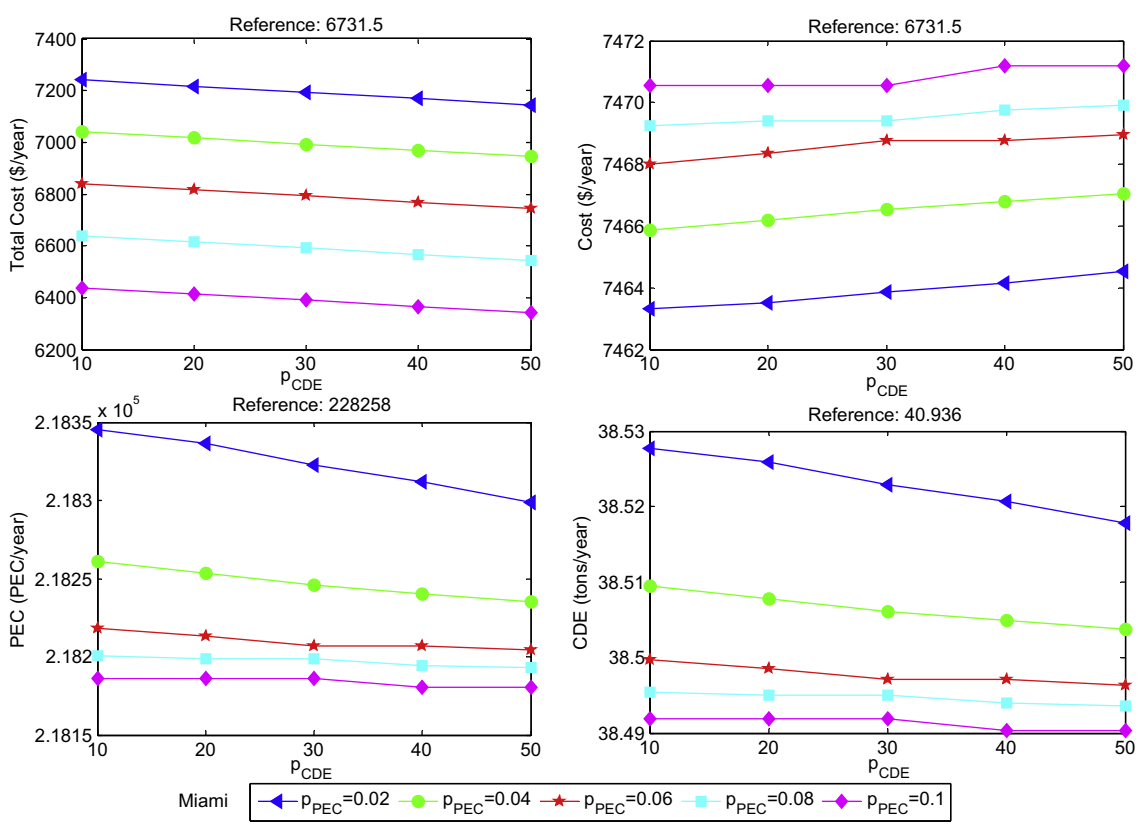

Fig. 8. Total cost, operational cost, PEC and CDE under different p_{PEC} and p_{CDE} values for Miami.

Table 6
Guidelines for selecting PEC incentive p_{PEC} and CDE incentive p_{CDE}.

	Columbus, MS	Minneapolis, MN	San Francisco, CA	Boston, MA	Miami, FL
f_{total}	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*s*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)
f_{cost}	p_{PEC} (*s*), p_{CDE} (*s*)	p_{PEC} (*s*), p_{CDE} (*l*)	p_{PEC} (*s*), p_{CDE} (*s*)	p_{PEC} (*s*), p_{CDE} (*s*)	p_{PEC} (*s*), p_{CDE} (*s*)
f_{PEC}	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*s*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)
f_{CDE}	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*s*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)	p_{PEC} (*l*), p_{CDE} (*l*)

For the city of Miami, it is observed that increasing PEC incentive p_{PEC} and/or increasing CDE incentive p_{CDE} will decrease total cost, PEC and CDE, and increase operational cost. It is worse to use CCHP economically compared to the conventional system for the city of Miami if PEC incentive is less than \$0.08 per kW h.

In summary, increasing PEC incentive p_{PEC} will decrease PEC and increase operational cost for all the five cities, and increasing CDE incentive p_{CDE} will decrease CDE for all the five cities. Based on the above analyses, the selection of PEC incentive p_{PEC} and CDE incentive p_{CDE} to minimize total cost, operational cost, PEC and CDE are summarized in Table 6. Please note "*s*" denotes small value, "*m*" denotes medium value, and "*l*" denotes large value. The small, medium, and large values are based on the values chosen for the sets p_{PEC} [0.02, 0.04, 0.06, 0.08, 0.1], and the sets p_{CDE} [10, 20, 30, 40, 50].

5. Conclusion

A stochastic multi-objective optimization model with probability constraints to consider the reliability of the CCHP system is proposed to optimize the CCHP operation strategy. The CCHP operation strategy for five different cities, Columbus, MS; Minneapolis, MN; San Francisco, CA; Boston, MA; and Miami, FL are studied based on operational cost, PEC and CDE. The proposed stochastic model is capable of deriving Pareto operation strategy for the CCHP system which could guarantee the derived operation strategy is reliable to satisfy the stochastic energy demand. Increasing the reliability level *r* for the probability constraints is equivalent to increasing the energy demand which will increase the operational cost, PEC and CDE for all the five cities. The Pareto frontier obtained at high reliability level is dominated by the Pareto frontier obtained at low reliability level. An incentive model is proposed to assist the decision makers selecting the CCHP operation strategy from the Pareto operation decisions. The analysis results show that increasing PEC incentive p_{PEC} will decrease PEC and increase operational cost for all the five cities, and increasing CDE incentive p_{CDE} will decrease CDE for all the five cities. In the future, the uncertainties in the system efficiency factors will be considered to study a more reliable operation strategy for CCHP system, and the dynamic pricing plans for natural gas and electricity will also be incorporated into the stochastic decision model.

Appendix A. stochastic decision model II

The procedure to simplify the stochastic decision model I is presented here. The constraint in Eq. (15) could be simplified as

$$F_{pgu}(t) = aE_{pgu}(t) + bS_{pgu}(t) \tag{56}$$

$$F_{pgu}(t) \leqslant MS_{pgu}(t) \tag{57}$$

where *M* is a big number which is commonly used in the integer programming model and is chosen as a large positive value. Replacing $Q_{rcv}(t) = \eta_{pgu\_th}F_{pgu}(t)$ and $E_{pgu}(t) = (F_{pgu}(t) - bS_{pgu}(t))/a$ in the PGU energy balance constraint (see Eq. (8)), we have

$$(1 + a\eta_{pgu\_th} - a)F_{pgu}(t) - bS_{pgu}(t) = -aEnergy_{loss\_pgu}(t) \leqslant 0 \tag{58}$$

Replacing $Q_{boiler}(t) = \eta_{boiler}F_{boiler}(t)$ in the boiler energy balance constraint (see Eq. (9)), we have

$$(\eta_{boiler} - 1)F_{boiler}(t) = -Energy_{loss\_boiler}(t) \leqslant 0 \tag{59}$$

Replacing $E_{pgu}(t) = (F_{pgu}(t) - bS_{pgu}(t))/a$ in the energy balance constraint for electric energy provided from PGU and EG (see Eq. (10)), we have

$$aE_{facility}(t) - aE_{grid}(t) - F_{pgu}(t) + bS_{pgu}(t) = -aE_{excess}(t) \leqslant 0 \tag{60}$$

Replacing $Q_{th\_cool}(t) = Q_{cool}(t)/\eta_{cool\_comp}$, $Q_{th\_heat}(t) = Q_{heat}(t)/\eta_{heat\_comp}$, $Q_{rcv}(t) = \eta_{pgu\_th}F_{pgu}(t)$ and $Q_{boiler}(t) = \eta_{boiler}F_{boiler}(t)$ in the energy balance constraint for thermal energy provided from PGU and boiler (see Eq. (11)), we have

$$Q_{cool}(t)/\eta_{cool\_comp} + Q_{heat}(t)/\eta_{heat\_comp} - \eta_{pgu\_th}F_{pgu}(t) - \eta_{boiler}F_{boiler}(t) = 0 \tag{61}$$

Replacing $Q_{th\_cool}(t) = Q_{cool}(t)/\eta_{cool\_comp}$ in the CHP cooling component energy balance constraint (see Eq. (12)), we have

$$(1 - 1/\eta_{cool\_comp})Q_{cool}(t) = -Energy_{loss\_c}(t) \leqslant 0 \tag{62}$$

Replacing $Q_{th\_heat}(t) = Q_{heat}(t)/\eta_{heat\_comp}$ in the CHP heating component energy balance constraint (see Eq. (13)), we have

$$(1 - 1/\eta_{heat\_comp})Q_{heat}(t) = -Energy_{loss\_h}(t) \leqslant 0 \tag{63}$$

Replacing $E_{excess}(t) + Energy_{loss\_total}(t) = E_{grid}(t) + F_{pgu}(t) + F_{boiler}(t) - E_{facility}(t) - Q_{cool}(t) - Q_{heat}(t)$ in the energy balance constraint for total energy required (see Eq. (1)), we have

$$P\{E_{facility}(t) + Q_{cool}(t) + Q_{heat}(t) \geqslant E_d(t) + Q_{cool\_d}(t) + Q_{heat\_d}(t)\} \geqslant r_1 \tag{64}$$

Constraints in Eqs. (59), (62), and (63) are satisfied since the efficiency factors η_{boiler}, η_{cool_comp}, and η_{heat_comp} are less than 1.

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